Public-key cryptography

Part 2: applications of public-key cryptography

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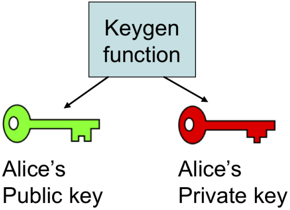
Outline

- Lecture 1: introduction to public-key cryptography
  - RSA encryption, signatures and DH key exchange
- Lecture 2: applications of public-key cryptography (this lecture)
  - Security models.
  - How to encrypt and sign securely with RSA. OAEP and PSS.
  - Public-key infrastructure. Certificates, SSL protocol.
- Lecture 3: cloud computing
  - How to delegate computation thanks to fully homorphic encryption
  - A fully homomorphic encryption scheme

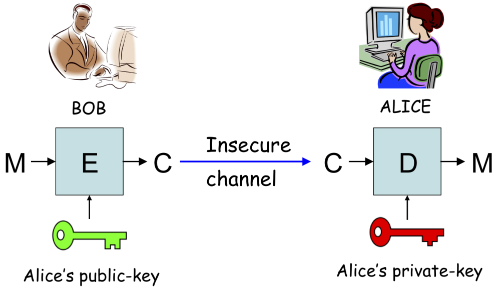
Public-key cryptography

- Invented by Diffie and Hellman in 1976. Revolutionized the field.
- Each user now has two keys
  - A public key
  - A private key
  - Should be hard to compute the private key from the public key.
- Enables:
  - Asymmetric encryption
  - Digital signatures
  - Key exchange, identification, and many other protocols.



Public-key encryption

- Public-key encryption (or asymmetric encryption)
  - Solves the key distribution issue

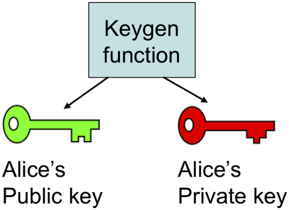


RSA

- Key generation:
  - Generate two large distinct primes p and q of same bit-size $k/2$, where k is a parameter.
  - Compute $n = p \cdot q$ and $\phi = (p-1)(q-1)$.
  - Select a random integer e such that $\gcd(e, \phi) = 1$
  - Compute the unique integer d such that

$$e \cdot d \equiv 1 \pmod{\phi}$$

    using the extended Euclidean algorithm.
  - The public key is (n, e).
  - The private key is d.



RSA encryption

- Encryption with public-key (n, e)
  - Given a message $m \in [0, n-1]$ and the recipent's public-key (n, e), compute the ciphertext:

$$c = m^e \bmod n$$

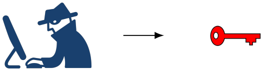
- Decryption with private-key d
  - Given a ciphertext c, to recover m, compute:

$$m = c^d \bmod n$$

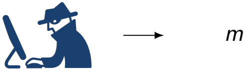
- Security is based on the hardness of factorization
  - Given $n = p \cdot q$, no known efficient algorithm to recover the primes p and q.
  - Public modulus n must be large enough: at least 1024 bits. 2048 bits is better.

Security models

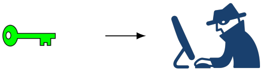
- To be rigorous when speaking about security, one must specify: the attacker's goal and the attacker's power.
- The attacker's goal
  - Does he need to recover the private key ?



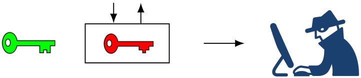
  - or only decrypt a particular ciphertext (or less) ?



- The attacker's power
  - Does he get only the user's public-key ?

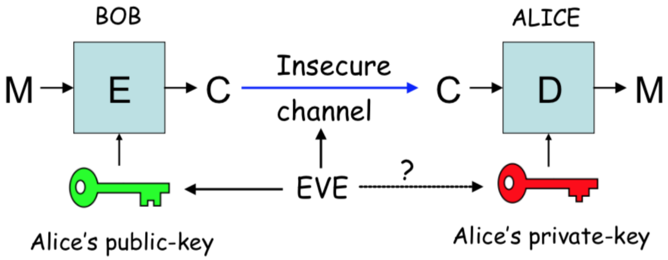


  - or more ?



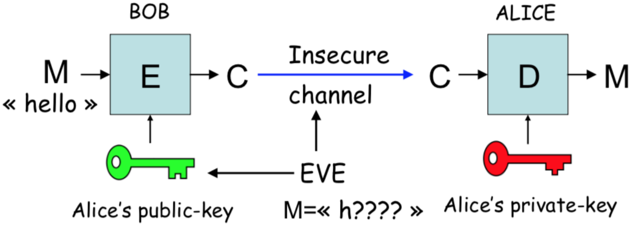
Attacker's goal

- One may think that the adversary's goal is always to recover the private key.
  - complete break
  - may be too ambitious in practice



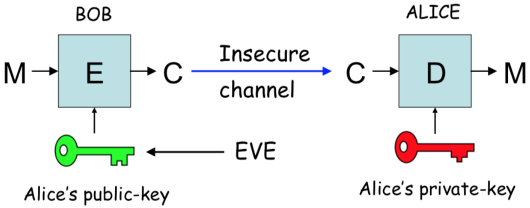
Attacker's goal

- More modest goal: being able to decrypt one ciphertext.
  - or recover some information about a plaintext (for example, the first character)



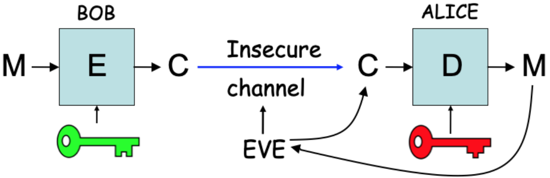
Attack model

- Specify the power of the attacker
- Public-key only attack: the attacker gets only the public-key
  - Called chosen plaintext attack (CPA) because the adversary can encrypt any plaintext of his choice.
  - Weakest adversary



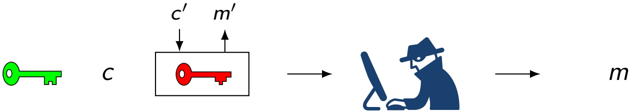
Chosen ciphertext attack (CCA)

- Most powerful attack
- The attacker can obtain decryption of messages of his choice
- May be realistic in practice
  - attacker gets access to a decryption machine
  - encryption algorithm used in a more complex protocol in which users can obtain decryption of chosen ciphertexts.



Chosen ciphertext attack against textbook RSA

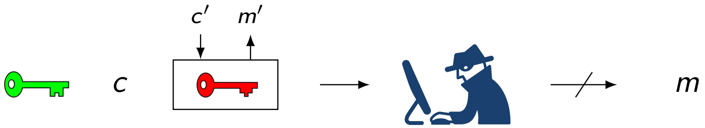
- Chosen-ciphertext attack:
  - Given ciphertext c to be decrypted
  - Generate a random r
  - Ask for the decryption of the random looking ciphertext $c' = c \cdot r^e \pmod{n}$
  - One gets $m' = (c')^d = c^d \cdot (r^e)^d = c^d \cdot r = m \cdot r \pmod{n}$
  - This enables to compute $m = m'/r \pmod{n}$



- Conclusion: do not use textbook RSA encryption !

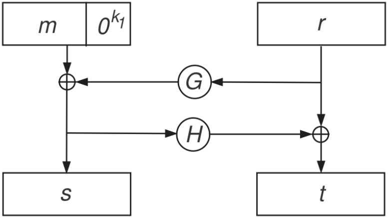
Strongest security notion for public-key encryption

- Indistinguishability under adaptive chosen ciphertext attack (IND-CCA2)
  - Formalized in 1991 by Rackoff et Simon
  - A ciphertext should give no information about the corresponding plaintext, even under an adaptive chosen-ciphertext attack.
  - Has become standard security notion for encryption.



OAEP

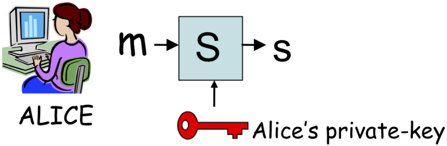
- OAEP (Bellare and Rogaway, E'94)
  - IND-CCA2, assuming that RSA is hard to invert.
  - PKCS #1 v2.1



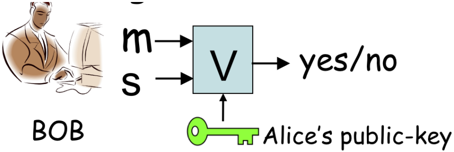
$$c = (s \| t)^e \bmod N$$

Digital signatures

- A digital signature σ is a bit string that depends on the message m and the user's public-key pk
  - Only Alice can sign a message m using her private-key sk



  - Anybody can verify Alice's signature of the message m given her public-key pk



The RSA signature scheme

- Key generation :
  - Public modulus: $N = p \cdot q$ where p and q are large primes.
  - Public exponent : e
  - Private exponent: d, such that $d \cdot e = 1 \pmod{\phi(N)}$
- To sign a message m, the signer computes :
  - $s = m^d \bmod N$
  - Only the signer can sign the message.
- To verify the signature, one checks that:
  - $m = s^e \bmod N$
  - Anybody can verify the signature

Attacks against textbook RSA signatures

- Given $\sigma_1 = (m_1)^d \bmod N$ and $\sigma_2 = (m_2)^d \bmod N$:
  - one can compute the signature of $m_1 \cdot m_2$ without knowing d:

$$\sigma = (m_1 \cdot m_2)^d = (m_1)^d \cdot (m_2)^d = \sigma_1 \cdot \sigma_2 \pmod{N}$$

- One cannot use plain RSA signature
  - Hash-and-sign paradigm: the message is first hashed

$$m \longrightarrow H(m) \longrightarrow 1001\ldots0101 \| H(m)$$

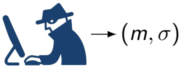
$$\downarrow$$

$$\sigma = \left(1001\ldots0101 \| H(m)\right)^d \pmod{N}$$

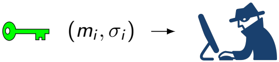
Attack scenario for signature schemes

- We must specify the adversary's goal and the adversary's power.
- Adversary's goal
  - Controlled forgery: the adversary can produce the signature of any message

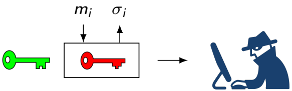
  - Existential forgery: the adversary can produce the signature of a (possibly meaningless) message



- Adversary's power
  - Known message attack: the adversary obtains a set of pairs message/signature

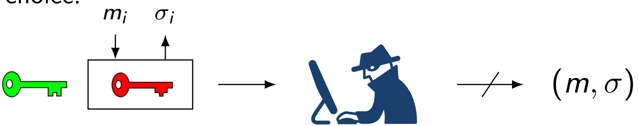


  - Chosen message attack: the adversary can obtain the signature of any message of his choice, adaptively.



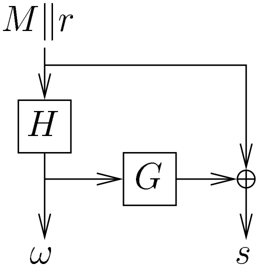
Strongest security notion for signature scheme

- Combines weakest goal with strongest adversary
- Existential unforgeability under an adaptive chosen message attack
  - Defined by Goldwasser, Micali and Rivest in 1988
  - It must be infeasible for an attacker to forge the signature of a message, even if he can obtain signatures of messages of his choice.



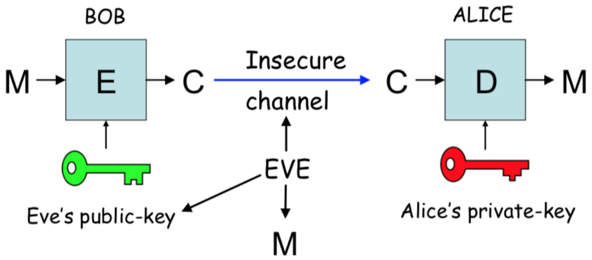
The PSS signature scheme

- PSS (Bellare and Rogaway, Eurocrypt'96)
  - IEEE P1363a and PKCS#1 v2.1.
  - 2 variants: PSS and PSS-R (message recovery)
  - Provably secure against chosen-message attacks, in the random oracle model.
  - PSS-R: $\mu(M, r) = \omega \| s$, $\sigma = \mu(M, r)^d \bmod N$



Public-key infrastructure

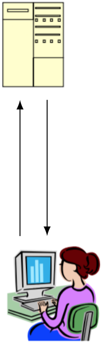
- Public-keys need to be authenticated
  - Bob needs to be sure that the public-key belongs to Alice.
  - Otherwise, impersonation attack



Public-key Infrastructure (PKI)

- A central authority binds public-keys to identities.
  - Public-key is stored in a certificate provided by the central authority
  - Used to prevent impersonation attack

Certificate authority (CA)



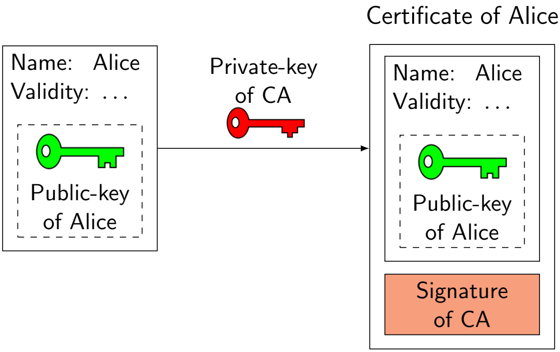
2) Alice authenticates and sends pk_{Alice}

3) Certificate (ID_{Alice}, pk_{Alice})

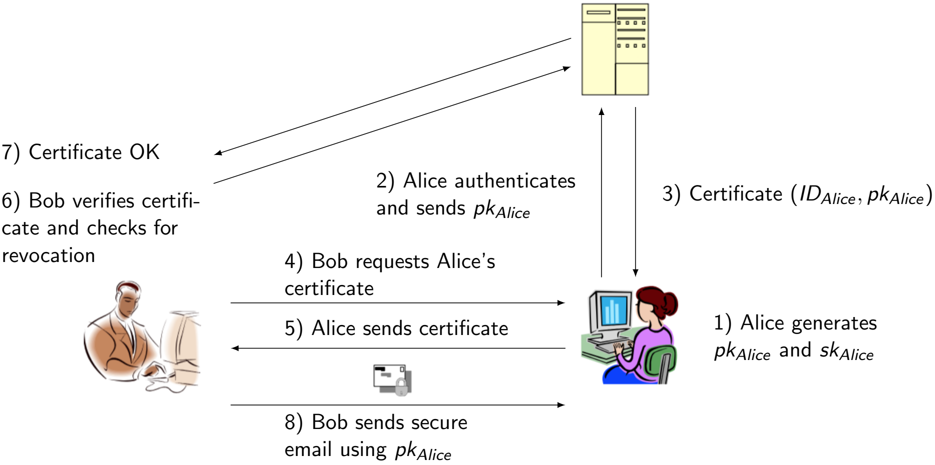
1) Alice generates pk_{Alice} and sk_{Alice}

Public-key certificate

- The signature of the certificate authority (CA) binds together a public-key with an identity in the certificate.
  - Proves ownership of a public-key
  - Bob can be sure that the public-key belongs to Alice by checking the signature using the CA public-key.
  - The CA is trusted by all participants.
  - Most common certificate standard: X.509

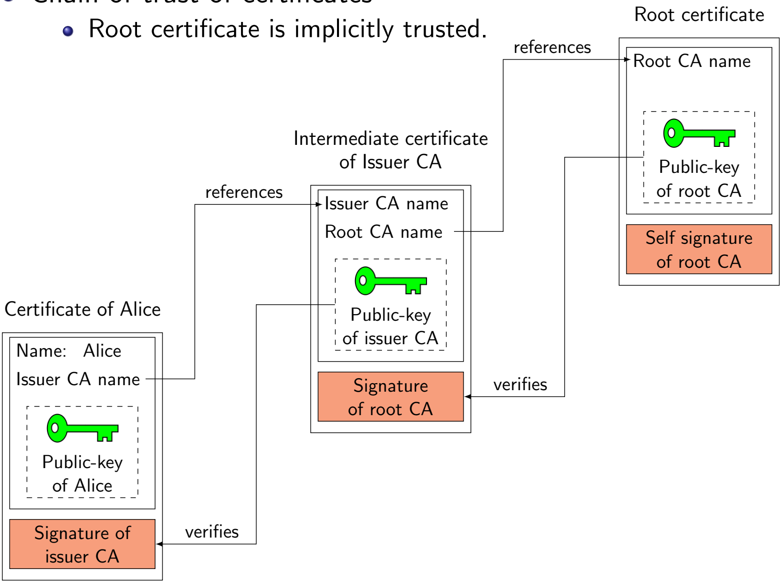


PKI encryption



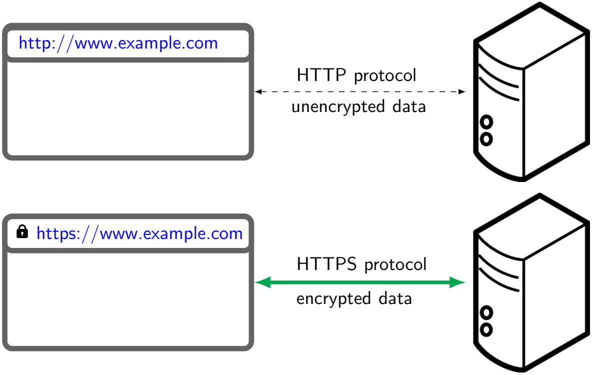
Hierarchy of certificates

- Chain of trust of certificates
  - Root certificate is implicitly trusted.



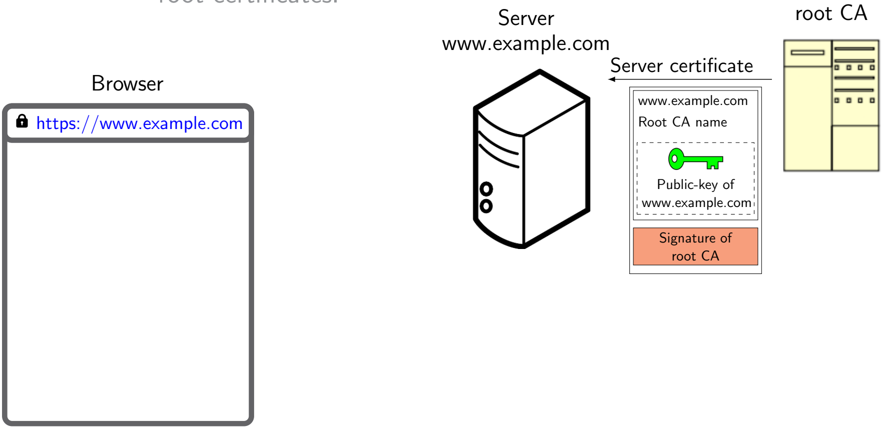
The HTTPS protocol

- Hypertext Transfer Protocol Secure (HTTPS) is a protocol for securely browsing the web.
  - Communication is encrypted using Transport Layer Security (TLS); formerly, Secure Sockets Layer (SSL).
  - Validates the authenticity of an HTTPS web server, and ensures confidentiality and integrity of communications.



Server authentication in HTTPS

- CA issues a certificate to the server to authenticate the server's public-key
  - The server expects the CA's certificate to be contained in most clients web browser.
  - One needs to trust the browser's publisher to include correct root certificates.



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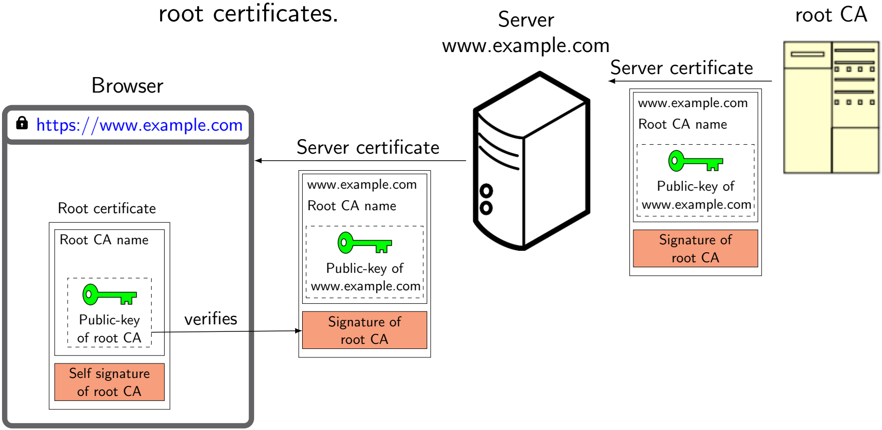
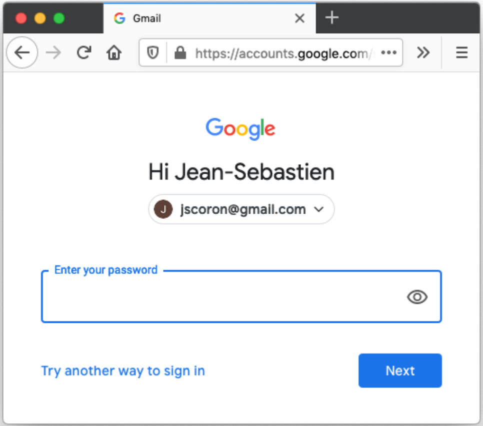


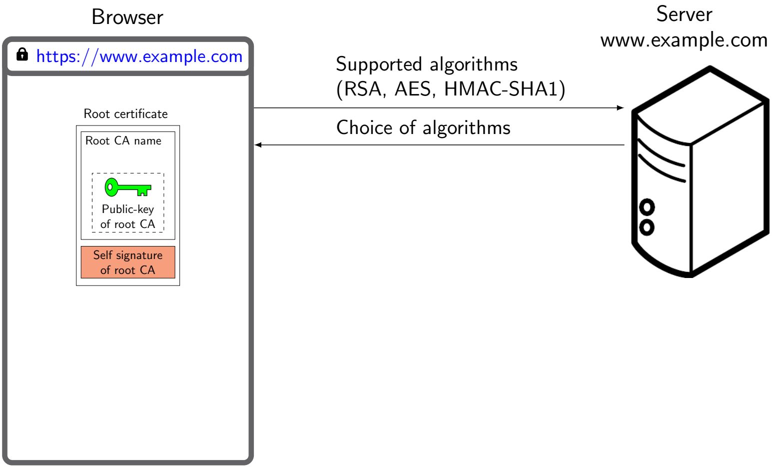
Client authentication in HTTPS

- Generally, only the server is authenticated
  - Mutual authentication requires a client certificate.
  - Most services use passwords to authenticate users, instead of client certificates.



The TLS protocol

- Three steps
  - Negotiation for algorithms used.
  - Certificate verification and PK encryption for session key.
  - Symmetric encryption for traffic encryption.



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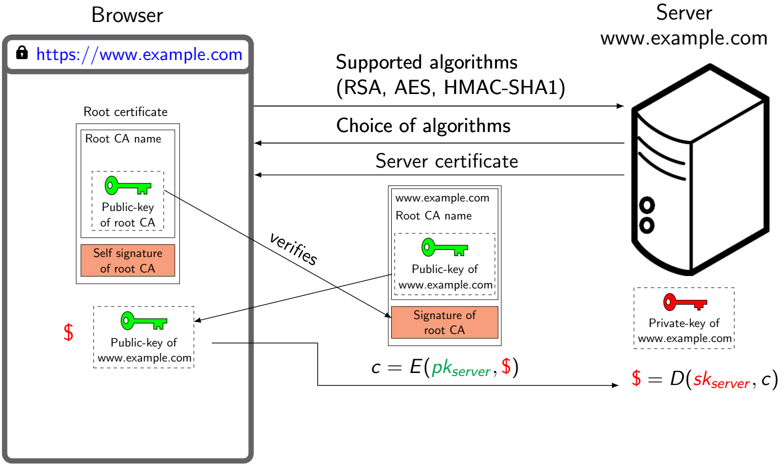

$c = E(pk_{server}, \$)$

$\$ = D(sk_{server}, c)$

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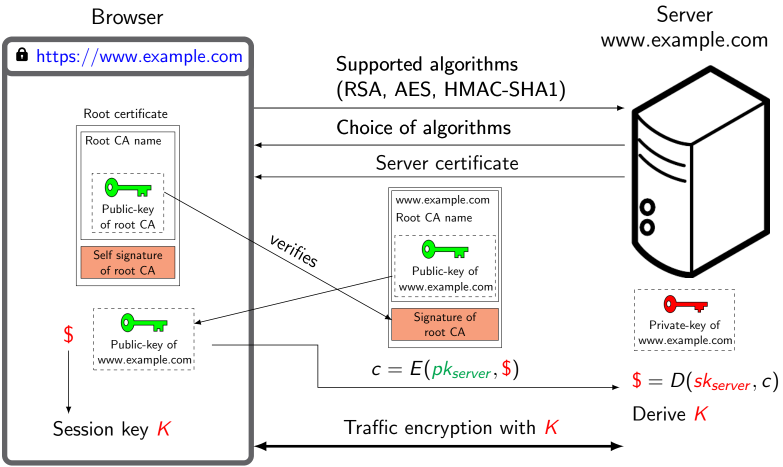

Credit card via HTTPS

- HTTPS only protects the credit card number during transit between the user's computer and the server
  - Does not protect against an attack on the server
- Attack on the server usually easier than interception in transit
  - Credit card number often saved in a database in merchant site
  - Attacks generally concentrate on the server and database