BICS Security 2

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BICS Security 2

- Description
  - This course allows students to obtain in-depth knowledge from a selection of areas in the field of information security.
- The course is divided into 3 parts:
  - Public-key cryptography (Jean-Sébastien Coron): 4 lectures
  - General cryptographic protocols (Peter Y. A. Ryan): 6 lectures
  - System security and trusted computation (Marcus Völp): 4 lectures
- Organization:
  - Lectures on Tuesdays, 11:00 - 12:30.
  - TDs on Fridays, 13:00 - 14:30.
- Grading
  - Homework (100 %)

Public-key cryptography

Part 1: introduction to public-key cryptography

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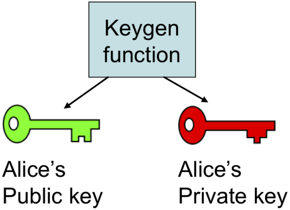
Outline

- Lecture 1: introduction to public-key cryptography (this lecture)
  - RSA encryption, signatures and DH key exchange
- Lecture 2: applications of public-key cryptography
  - Security models.
  - How to encrypt and sign securely with RSA. OAEP and PSS.
  - Public-key infrastructure. Certificates, SSL protocol.
- Lecture 3: cloud computing
  - How to delegate computation thanks to fully homorphic encryption
  - A fully homomorphic encryption scheme

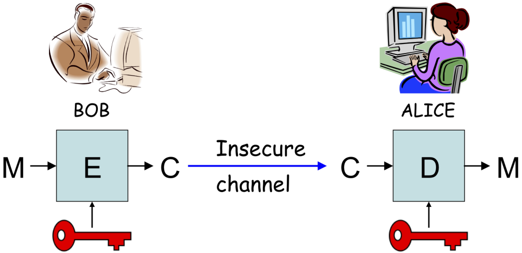
Public-key cryptography

- Invented by Diffie and Hellman in 1976. Revolutionized the field.
- Each user now has two keys
    - A public key
    - A private key
    - Should be hard to compute the private key from the public key.
- Enables:
    - Asymmetric encryption
    - Digital signatures
    - Key exchange, identification, and many other protocols.



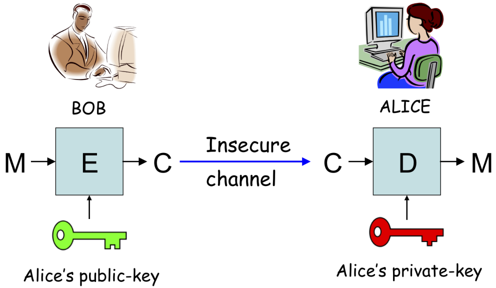
Key distribution issue

- Symmetric cryptography
  - Problem: how to initially distribute the key to establish a secure channel ?



Public-key encryption

- Public-key encryption (or asymmetric encryption)
    - Solves the key distribution issue

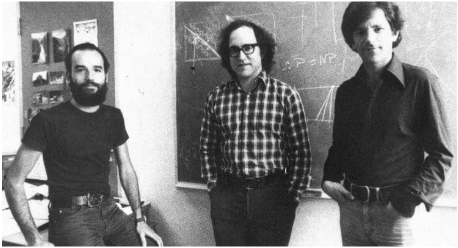


Analogy: the mailbox

- Bob wants to send a letter to Alice
  - Bob obtains Alice's adress
  - Bob puts his letter in Alice's mailbox
  - Alice opens her mailbox and read Bob's letter.
- Properties of the mailbox
  - Anybody can put a letter in the mailbox
  - Only Alice can open her mailbox

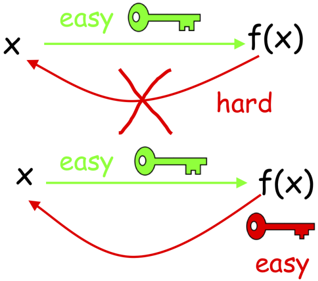
The RSA algorithm

- The RSA algorithm is the most widely-used public-key encryption algorithm
  - Invented in 1977 by Rivest, Shamir and Adleman.
  - Implements a trapdoor one-way permutation
  - Used for encryption and signature.
  - Widely used in electronic commerce protocols (SSL), secure email, and many other applications.



Trapdoor one-way permutation

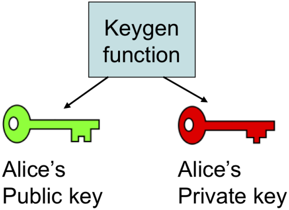
- Trapdoor one-way permutation
  - Computing $f(x)$ from x is easy
  - Computing x from $f(x)$ is hard without the trapdoor



- Public-key encryption
  - Anybody can compute the encryption $c = f(m)$ of the message m.
  - One can recover m from the ciphertext c only with the trapdoor.

RSA

- Key generation:
  - Generate two large distinct primes p and q of same bit-size $k/2$, where k is a parameter.
  - Compute $n = p \cdot q$ and $\phi = (p-1)(q-1)$.
  - Select a random integer e such that $\gcd(e, \phi) = 1$
  - Compute the unique integer d such that
    $$e \cdot d \equiv 1 \pmod{\phi}$$
    using the extended Euclidean algorithm.
  - The public key is (n, e).
  - The private key is d.



RSA encryption

- Encryption with public-key (n, e)
  - Given a message $m \in [0, n-1]$ and the recipent's public-key (n, e), compute the ciphertext:

$$c = m^e \bmod n$$

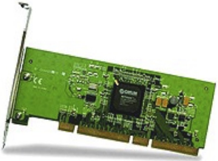
- Decryption with private-key d
  - Given a ciphertext c, to recover m, compute:

$$m = c^d \bmod n$$

- Message encoding
  - The message m is viewed as an integer between 0 and $n-1$
  - One can always interpret a bit-string of length less than $\lfloor \log_2 n \rfloor$ as such a number.

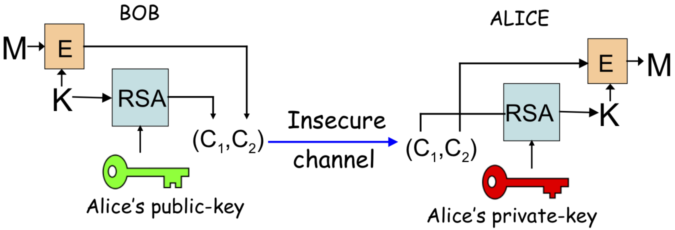
Implementation of RSA

- Required: computing with large integers
    - more than 1024 bits.
- In software
    - big integer library: GMP, NTL
- In hardware
    - Cryptoprocessor for smart-card
    - Hardware accelerator for PC.



Speed of RSA

- RSA much slower than AES and other secret key algorithms.
- To encrypt long messages
  - encrypt a symmetric key K with RSA
  - and encrypt the long message with K

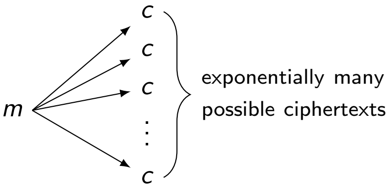


Security of RSA

- The security of RSA is based on the hardness of factoring.
  - Given $n = p \cdot q$, it should be difficult to recover p and q.
  - No efficient algorithm is known to do that. Best algorithms have sub-exponential complexity.
  - Factoring record (2020): a 829-bit RSA modulus n.
  - In practice, one uses at least 1024-bit RSA moduli.
- However, there are many other lines of attacks.
  - Attacks against textbook RSA encryption
  - Low private / public exponent attacks
  - Implementation attacks: timing attacks, power attacks and fault attacks

Elementary attacks

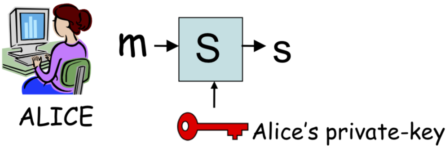
- Textbook RSA encryption: dictionary attack
  - If only two possible messages m_0 and m_1, then only $c_0 = (m_0)^e \bmod N$ and $c_1 = (m_1)^e \bmod N$.
  - $\Rightarrow$ encryption must be probabilistic.



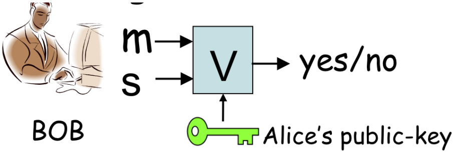
- Example: PKCS#1 v1.5 (1993)
  - $\mu(m) = 0002\|r\|00\|m$
  - $c = \mu(m)^e \bmod N$
  - Still insufficient (Bleichenbacher's attack, 1998)

Digital signatures

- A digital signature σ is a bit string that depends on the message m and the user's public-key pk
  - Only Alice can sign a message m using her private-key sk



  - Anybody can verify Alice's signature of the message m given her public-key pk



Digital signature

- A digital signature provides:
    - Authenticity: only Alice can produce a signature of a message valid under her public-key.
    - Integrity: the signed message cannot be modified.
    - Non-repudiation: Alice cannot later claim that she did not sign the message

The RSA signature scheme

- Key generation :
    - Public modulus: $N = p \cdot q$ where p and q are large primes.
    - Public exponent : e
    - Private exponent: d, such that $d \cdot e = 1 \mod \phi(N)$
- To sign a message m, the signer computes :
    - $s = m^d \mod N$
    - Only the signer can sign the message.
- To verify the signature, one checks that:
    - $m = s^e \mod N$
    - Anybody can verify the signature

Hash-and-sign paradigm

- There are many attacks on basic RSA signatures:
  - Existential forgery: $r^e = m \pmod N$
  - Chosen-message attack: $(m_1 \cdot m_2)^d = m_1^d \cdot m_2^d \pmod N$
- To prevent from these attacks, one usually uses a hash function. The message is first hashed, then padded.

$$\begin{tikzcd} m \arrow[r] & H(m) \arrow[r] & 1001\ldots0101\|H(m) \arrow[d] \\ & & \sigma = \left(1001\ldots0101\|H(m)\right)^d \bmod N \end{tikzcd}$$

- Example: PKCS#1 v1.5 (1993)

$$\mu(m) = 0001\ \mathtt{FF....FF00}\|c_{\mathsf{SHA}}\|\mathsf{SHA}(m)$$

  - The signature is then $\sigma = \mu(m)^d \bmod N$

Other signature schemes

- Digital Signature Algorithm (DSA) (1991)
  - Digital Signature Standard (DSS) proposed by NIST, specified in FIPS 186.
  - Variant of Schnorr and ElGamal signature schemes
  - Security based on the hardness of discrete logarithm problem.
  - Public-key: $y = g^{x} \bmod p$
  - Signature: (r, s), where $r = (g^k \bmod p) \bmod q$ and $s = k^{-1}(H(m) + x \cdot r) \bmod p$, where $k \overset{\$}{\leftarrow} \mathbb{Z}_q$
- ECDSA: a variant of DSA for elliptic-curves
  - Shorter public-key than DSA (160 bits instead of 1024 bits)
  - Used in Bitcoin to ensure that funds can only be spent by their rightful owners.

Diffie-Hellman key-exchange protocol

- Public parameters: g and p

Bob

Alice

$$B = g^{b}\ [p] \qquad \xrightarrow{B} \qquad A = g^{a}\ [p]$$

$$\xleftarrow{A}$$

$$K_B = A^b = (g^a)^b = g^{ab}\ [p] \qquad K_A = B^a = (g^b)^a = g^{ba}\ [p]$$

$$K_B = K_A$$

Security of Diffie-Hellman

- Based on the hardness of the discrete-log problem:
  - Given $A = g^a \pmod{p}$, find a
  - No efficient algorithm for large prime p.
- No authentication
  - Vulnerable to the man in the middle attack

Diffie-Hellman: man in the middle attack

Bob

Alice

$$B = g^b \ [p] \qquad \xrightarrow{\quad B \quad} \qquad A = g^a \ [p]$$

$$\xleftarrow{\quad A \quad}$$

$$K_B = A^b = g^{ab} \ [p] \qquad\qquad K_A = B^a = g^{ba} \ [p]$$

$$K_B = K_A$$

Diffie-Hellman: man in the middle attack

Eve

Bob

Alice

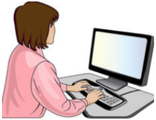
$$B = g^{b}\ [p] \quad \xrightarrow{\quad B \quad} \quad A = g^{a}\ [p]$$

$$\xleftarrow{\quad A \quad}$$

$$K_B = A^{b} = g^{ab}\ [p] \qquad K_A = B^{a} = g^{ba}\ [p]$$

$$K_B = K_A$$

Diffie-Hellman: man in the middle attack



Eve

Bob

$B = g^b \; [p]$

Alice

$A = g^a \; [p]$

$\xrightarrow{B}$

$\xrightarrow{B' = g^{b'} \; [p]}$

$\xleftarrow{A' = g^{a'} \; [p]}$

$\xleftarrow{A}$

$K'_B = (A')^b = g^{a'b} \; [p]$

$K'_A = (B')^a = g^{b'a} \; [p]$

$$K'_B = B^{a'} \; [p]$$
$$K'_A = A^{b'} \; [p]$$

Security of Diffie-Hellman

- Based on the hardness of the discrete-log problem:
  - Given $A = g^a \pmod{p}$, find a
  - No efficient algorithm for large prime p.
- No authentication
  - Vulnerable to the man in the middle attack
- Authenticated key exchange
  - Using a PKI. Alice and Bob can sign A and B
  - Password-authenticated key-exchange IEEE P1363.2

Lessons from the past

- Cryptography is a permanent race between construction and attacks
    - but somehow this has changed with modern cryptography and security proofs.
- Security should rely on the secrecy of the key and not of the algorithm
    - Open algorithms enables open scrutiny.

- Note: installation of Sage
    - Install Sage `https://www.sagemath.org`
    - Run a Jupyter notebook
      $ sage -n jupyter